Superfocusing by nanoshells

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Received November 14, 2008; revised February 10, 2009; accepted February 13, 2009; posted March 3, 2009 (Doc. ID 104133); published March 25, 2009

Recently researchers demonstrated, both theoretically and in the microwave range experimentally, subwavelength focusing of evanescent waves by patterned plates. The present Letter extends these ideas and the design procedure to scatterers of arbitrary shapes and to the optical range of wavelengths. The analytical study is supported by numerical results. The most intriguing feature of the proposed design is that, in the framework of classical electrodynamics of continuous media, focusing can in principle be arbitrarily sharp, subject to the constraints of fabrication. © 2009 Optical Society of America

OCIS codes: 310.6628, 160.4236, 350.4238, 240.6680, 250.5403, 050.1970.

Subwavelength focusing that circumvents the Abbe diffraction limit in optics has been a very active area of research, with a multitude of approaches explored in the literature: negative-index lenses and guides, plasmonic particles and cascades, superoscillations, time-reversal techniques, and others ([1–6] and references therein). Recently Merlin and Grbic *et al.* [7,8] (see also [9]) showed, both theoretically and in the microwave range experimentally, that patterned (gratinglike) plates produce subwavelength focusing of evanescent waves if the pattern contains significantly different spatial scales of variation. Conceptually, these patterns are related to surface profiles of near-field optical holography [10,11].

This Letter shows that the ideas of near-field focusing can be extended to nanoshells and, further, to scatterers of arbitrary shape. The most intriguing feature is "superfocusing" that can in principle be arbitrarily sharp and strong, subject to the constraints of fabrication and availability of materials with desired values of the dielectric permittivity ϵ. (It is also tacitly assumed that the size of the system is sufficiently large for electrodynamics of continuous media to be applicable [6].)

Let an incident plane wave with a frequency ω be scattered by a thin plasmonic and/or dielectric shell. In general, the shape of the scatterer may vary; its thickness and dielectric permittivity may be chosen judiciously, with the ultimate goal of nanofocusing the wave at a given spot. In practical applications, the role of the scatterer can be played, e.g., by the apex of an optical tip, the active part of an optical sensor, or by a nanoantenna.

Even though the ideas are general and could be applied in 3D electrodynamic analysis and design, let us start for maximum simplicity with the 2D case of a cylindrical scatterer (particle) of radius r_{cyl} (Fig. 1).

Several ways of formulating this electromagnetic field problem are available. First, one needs to distinguish full electrodynamic analysis versus quasi-static (QS) approximations valid for dimensions much smaller than the wavelength. The QS treatment has limitations, as it does not account for wave effects and short-range surface plasmon modes [12]. However, to fix ideas and for the sake of algebraic simplicity this Letter adopts the QS approximation. Full-wave analysis in 2D is analogous but involves Bessel/Hankel functions and their derivatives.

Second, the electrodynamic problem may be formulated in terms of the electric or magnetic field (convenient for s and p polarizations, respectively). Alternatively, two dual formulations are also available under the QS approximation: via the electric scalar potential and via the stream function $\mathbf{\Psi}$, $\nabla \times \mathbf{\Psi} = \epsilon \mathbf{E}$ [13]; see [14,15] for some interesting implications of this duality.

Let us endeavor to achieve sharp focusing with respect to the polar angle ϕ at a certain radius $r_f > r_{\text{cyl}}$. For p polarization, let

$$H_{\text{out}}(r_f, \phi) = H_f g(\phi - \phi_f), \tag{1}$$

where H_{out} is the magnetic field outside the shell, H_f is a scaling parameter, ϕ_f is a given angle, and $g(\phi)$ is a desired distribution of the field. For example, choosing $g(\phi)$ or, alternatively, $g'(\phi)$, as a sharp Gaussian peak will result in the corresponding peak

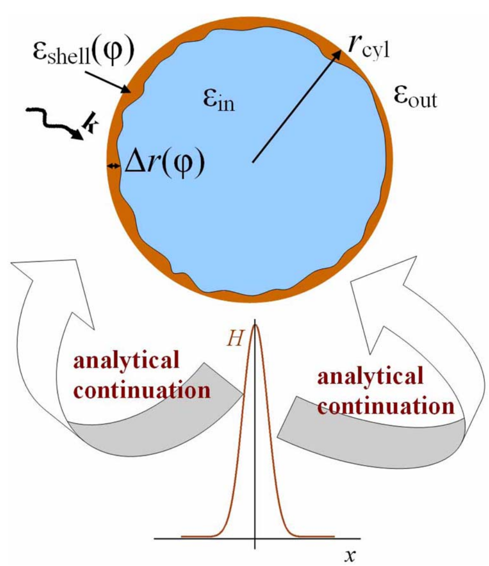

Fig. 1. (Color online) A thin shell (coating) can be capable of focusing light to an arbitrarily narrow spot if the angular variation of its dielectric function and/or thickness are judiciously chosen. See text for details.

in the magnetic field or the radial component of the electric field, respectively.

This local behavior of the field can be extended, by analytical continuation, to the whole region outside the scattering shell. One way of doing so is by expanding the field into cylindrical harmonics whose coefficients are found from the Fourier transform of g. This analytical continuation generates a field distribution on the surface of the shell, and one needs to find parameters that would produce such a surface distribution.

More specifically, in the QS limit the field inside the shell can be expanded into cylindrical harmonics:

$$H_{\text{in}}(r,\phi) = \sum_{n=-\infty}^{\infty} a_n r^{|n|} \exp(in\phi), \qquad r \leqslant r_{\text{cyl}}, \tag{2}$$

where a_n are some coefficients. Outside of the cylinder ($r > r_{\text{cyl}}$) the magnetic field can be represented as the sum of the incident field and the scattered field, $H_{\text{out}}=H_{\text{INC}}+H_{\text{s}}$. The incident field under the QS approximation is linear with respect to r: $H_{\text{INC}}=H_0 + h_0 r \cos(\phi-\phi_0)$, where h_0 is a coefficient. The constant H_0 does not affect the results and is for brevity omitted in the remainder. The scattered field is

$$H_{\text{s}}(r,\phi) = \sum_{n=-\infty}^{\infty} c_n r^{-|n|} \exp(in\phi), \tag{3}$$

where c_n are coefficients to be determined.

The boundary conditions across the coating are as follows. If the coating is nonmagnetic [16] and thin, the magnetic flux passing through it can be neglected, and consequently the tangential component E_ϕ of the electric field is assumed to be continuous across the coating: $E_{\phi,\text{out}}=E_{\phi,\text{in}}$. Since $E_\phi = (i\omega\epsilon)^{-1}\partial H/\partial r$ [with the $\exp(-i\omega t)$ complex phasor convention], we have

$$\frac{\partial H_{\text{in}}}{\partial r} = \frac{\epsilon_{\text{in}}}{\epsilon_{\text{out}}}\frac{\partial H_{\text{out}}}{\partial r}, \tag{4}$$

which links the coefficients a_n, c_n as follows:

$$a_n = -c_n r_{\text{cyl}}^{-2|n|}\epsilon_{\text{in}}\epsilon_{\text{out}}^{-1}, \qquad n \neq \pm 1, \tag{5}$$

$$a_{\pm 1} = \epsilon_{\text{in}}\epsilon_{\text{out}}^{-1}\left(-c_{\pm 1} r_{\text{cyl}}^{-2} + \frac{1}{2}h_0 \exp(\mp i\phi_0)\right). \tag{6}$$

The jump of the magnetic field is *not* neglected, as it corresponds to a current layer that may be appreciable if $|\epsilon_{\text{shell}}|$ is large (e.g. for plasmonic materials):

$$H_{\text{out}} - H_{\text{in}} = i\omega\epsilon_{\text{shell}}(\phi)E_\phi \Delta r(\phi). \tag{7}$$

Again expressing E_ϕ in terms of H, one arrives at

$$H_{\text{out}} - H_{\text{in}} = \frac{\epsilon_{\text{shell}}(\phi)}{\epsilon_{\text{in/out}}}\Delta r(\phi)\frac{\partial H_{\text{in/out}}}{\partial r}, \tag{8}$$

where the "in/out" subscript in the right-hand side encompasses two equally valid expressions: one with ϵ_{in} and H_{in} and another one with ϵ_{out} and H_{out}.

From superfocusing at a point to fields on the cylinder. Let the desired field distribution around a certain focusing point $r=r_f>r_{\text{cyl}}$, $\phi=\phi_f$ be given by Eq. (1). This defines the following expansion coefficients c_n:

$$c_n = \frac{H_f}{r_f^{|n|}}\tilde{g}_n, \qquad n \neq \pm 1, \quad c_{\pm 1} = \frac{H_f}{r_f} - \frac{1}{2}h_0 r_f \exp(\mp i\phi_0), \tag{9}$$

where $\tilde{g}_n=(2\pi)^{-1}\int_0^{2\pi} g(\phi-\phi_f)\exp(in\phi)d\phi$ are the Fourier coefficients. One can now evaluate a_n from Eqs. (5) and (6), then find the jump of the magnetic field, and consequently, from Eq. (8), the required parameter

$$\epsilon_{\text{shell}}(\phi)\Delta r(\phi) = \frac{\epsilon_{\text{in}}[H_{\text{out}}(r_{\text{cyl}}) - H_{\text{in}}(r_{\text{cyl}})]}{\partial H_{\text{in}}(r_{\text{cyl}})/\partial r}. \tag{10}$$

This result is valid for thin nonmagnetic shells and also holds for the full electrodynamic problem, although the coefficients a_n, c_n implicit in Eq. (10) are different in the wave case [17].

The product of the permittivity and thickness of the shell in Eq. (10) reflects the impedancelike nature of the thin-shell boundary condition, $Z \equiv E_\phi/(H_{\text{out}}-H_{\text{in}}) = i/(\omega\epsilon_{\text{shell}}\Delta r)$, has the physical meaning of impedance.

The overall procedure can be summarized as follows. (i) Choose r_{cyl}, r_f, and the maximum number N of cylindrical harmonics. (ii) Compute c_n from Eq. (9), for $|n| \leqslant N$. (iii) Find a_n from Eqs. (5) and (6). (iv) Compute $H_{\text{out}}(r_{\text{cyl}})$, $H_{\text{in}}(r_{\text{cyl}})$, and $\partial H_{\text{in}}(r_{\text{cyl}})/\partial r$ using the cylindrical harmonic expansion with the coefficients a_n, c_n found in (ii), (iii). (v) Find $\epsilon_{\text{shell}}(\phi)\Delta r(\phi)$ from Eq. (10).

Changing the desired field variation $g(\phi)$ and the amplitude H_f, one obtains a very rich variety of solutions for the shell parameters [Eq. (10)]. To illustrate some of the possibilities, consider the angular distribution of $\epsilon_{\text{shell}}\Delta r/r_{\text{cyl}}$ at $r=r_f=1.2r_{\text{cyl}}$. For relatively small values of H_f, the sign of ϵ_{shell} varies (Fig. 2, $H_f=2$, $\Delta\phi=2\pi/16$, $N=12$). Higher values of H_f require a strong plasmonic resonance with $\epsilon_{\text{shell}}(\phi)$ consistently negative (Fig. 3, $H_f=10$, $N=20$, and $\Delta\phi = 2\pi/18$). Here the derivative $g'(\phi)$ was chosen as a Gaussian peak $g'(\phi)=\exp(-\phi^2/\Delta\phi^2)$, where parameter $\Delta\phi$ controls the width of the peak. This angular

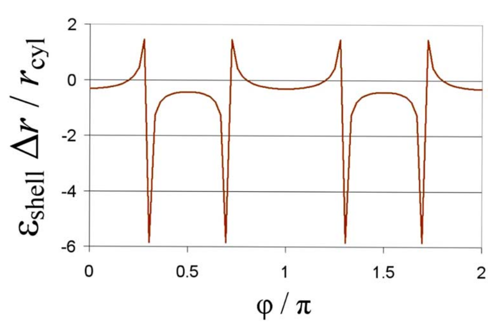

Fig. 2. (Color online) $\epsilon_{\text{shell}}\Delta r/r_{\text{cyl}}$ versus angle for $H_f=2$, $r_f=1.2r_{\text{cyl}}$ and $\Delta\phi=2\pi/16$, $N=12$ harmonics.

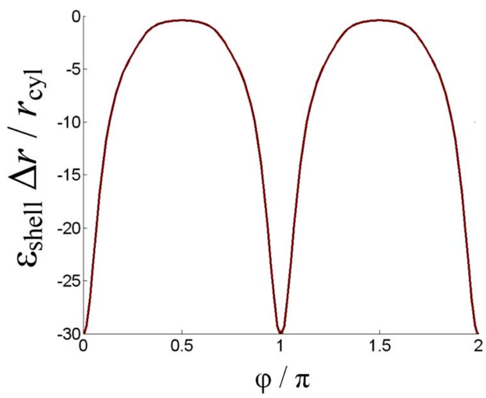

Fig. 3. (Color online) $\epsilon_{\text{shell}}\Delta r/r_{\text{cyl}}$ vs. angle for $H_f=10$, $r_f=1.2r_{\text{cyl}}$ and $\Delta\phi=2\pi/18$, $N=20$ harmonics.

distribution of the magnetic field leads to a respective peak in $E_r(\phi)$ (Fig. 4). The fields were computed semianalytically using Eq. (3) and also numerically using finite-difference (FD) analysis on regular polar grids. The semianalytical solution, valid in the absence of losses, is close to the numerical one (Fig. 4). The discrepancy is due primarily to the finite thickness of the shell $\Delta r=0.02r_{\text{cyl}}$ in FD simulations, and to the limited number of cylindrical harmonics in the semianalytical treatment. Losses affect the amplitude of the peak but not its sharpness ($\epsilon''_{\text{shell}}=0.1$, empty squares in Fig. 4). Convergence of FD results was verified by running the simulation on different grids (e.g., almost coinciding dashed and dotted curves in Fig. 4).

In summary, superfocusing of light by nanoshells, with the sharpness of the focus in principle unlimited, has been demonstrated analytically and numerically. The shell is designed in two stages: (i) analytical continuation of the desired behavior of the field at the focus to the boundary of the scatterer and (ii) finding the angular distribution of the permittivity and thickness of the shell that would produce the required field on the surface. Recently proposed focusing by superoscillations [3] also falls into this framework, with the waveform at the focus cleverly chosen to contain only traveling-wave components.

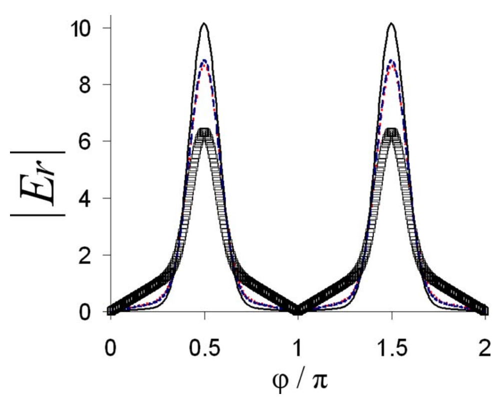

Fig. 4. (Color online) E_r versus angle at the radius of the focus $r_f=1.2r_{\text{cyl}}$. $H_f=10$, $N=20$, $\Delta\phi=2\pi/18$. Solid curve, semianalytical solution. Coinciding dashed and dotted curves, no losses ($\epsilon''=0$), FD grids $n_r\times n_\phi=150\times480$ and 300×720. Empty squares, $\epsilon''=0.1$, grid 150×480.

The procedure can be generalized to arbitrary shapes of the shell and to 3D by still using cylindrical/spherical harmonic expansions in the outside region (as in T-matrix methods [18]) and numerical methods (e.g., the finite-element method) inside. Further improvement of the focusing effects should be possible with advanced numerical optimization techniques (e.g. adaptive goal-oriented finite-element analysis [19]) that will treat not only the physical properties of the shell but also its geometric shape as adjustable parameters. Finally, it would be interesting to explore superfocusing of plasmon polaritons propagating on judiciously designed patterned surfaces [3].

I thank N. I. Zheludev, S. I. Bozhevolnyi, and the anonymous reviewer for very helpful comments.

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