

# A Comprehensive Design Methodology for Switched Reluctance Machines

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**Abstract**—The description of a comprehensive design methodology for switched reluctance machines (SRMs) is reported here. The magnetic properties of the iron, the number of phases, and the number of poles per phase all have a nonlinear effect on an SRM's performance. These effects, along with the sizing of the machine envelope and internal dimensions, make the machine design an insight-intensive effort. Maximization of torque density, power output, efficiency, speed range, and first critical speed and minimization of torque ripple, temperature rise, acoustic noise, and overall cost are among the many design objectives and critical issues that must be addressed during the design process. A design methodology that maximizes the desired features and minimizes the unwanted effects is presented here. Static and dynamic system-level simulations and finite-element analysis have been carried out for a four-phase 8/6 1.0-kW SRM as a design example to support the efficacy of the proposed design procedure.

**Index Terms**—Switched reluctance machines.

## NOMENCLATURE

### Dimensions,<sup>1</sup> Configuration, and Parameters

$A_{pe}, A_{pm}$	Electrical and mechanical area product, respectively.
$A_{wn}, K$	Net window area for each coil side and window factor.
$A_{wire}$	Cross-sectional area of each wire in the pole winding.
$B_{sat}, g$	Saturation flux density and air-gap length, respectively.
$\beta_s, \beta_r$	Stator and rotor pole-arc width for each repetition.
$\Delta T$	Temperature rise halfway along the $L_{stk}$ on the coil side.
$f_{m(=0,2,4)}$	Circumferential mode frequency for mode $m = 0, 2, \text{ and } 4$ .

$h_s, h_r$	Stator and rotor pole height, respectively.
$I_{plp}, J_{rms}$	Peak phase current and rms current density, respectively.
$K_L, S_{tf}$	Inductance overlap ratio and stacking factor.
$l_{r1}$	Spacing between unaligned stator and rotor pole tips.
$n_{ser}, n_{par}$	Number of series and parallel paths in the winding.
$N_{ph}, N_{rep}$	Number of phases and repetition, respectively.
$L_{stk}, N_p$	Stack length and number of turns per pole, respectively.
$n_c, P_{max}$	First critical speed and maximum rated power.
$P, Pf$	Pole pitch = $\pi \cdot R_{out} \cdot N_{ph}^{-1} \cdot N_{rep}^{-1}$ and power factor.
$R_{shft}, R_{ry}$	Shaft radius and radius to rotor yoke, respectively.
$R_g, R_{sy}$	Radius to rotor pole tips and the radius to stator yoke.
$R_{out}, R_{ph}$	Machine outer radius and resistance of each phase.
$\rho_m, T_s$	Material density and hoop stress, respectively.
$T_{e\rho}, T_{rated}$	Maximum rotor torque density and rated torque.
$t_s, t_r$	Stator and rotor lateral pole width, respectively.
$\theta_c^\lambda$	Critical rotor position for flux density.
$\theta_c^i$	Critical rotor position for current.
$W_t, V_{bus}$	Total weight of the machine and bus voltage.
$\omega_b, \omega_{mrated}$	Base and maximum rated speed of the machine.
$y_s, y_r$	Thickness of the stator and rotor yoke, respectively.
$\alpha_{rms}$	Ratio of rms to peak phase currents.
$\alpha_u$	Ratio of unaligned to aligned flux linkage.
$\eta$	Efficiency of the machine.

### Design Ratios

$\alpha_{hrtr1}, \alpha_{hrhs}, \alpha_{gyrs}$	Ratios of $h_r/l_{r1}$ , $h_r/h_s$ , and $y_r/y_s$ , respectively.
$\alpha_n, \alpha_{RgRout}, \alpha_{tsP}$	Ratios of $y_s/t_s$ , $R_g/R_{out}$ , and $t_s/P$ , respectively.

## I. INTRODUCTION

THE switched reluctance machine (SRM) is a viable contestant for a number of applications that require high reliability, wide speed range at constant power, low manufacturing cost, and fast dynamic response. Like all other machines, the SRM has a unique set of characteristics that make it suitable for certain applications and unsuitable for others. The degree of suitability of an SRM for any application depends on the application's requirements and objectives. The specific weighting

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<sup>1</sup>Dimensions are shown in Fig. 2.

of the numerous design objectives, common to all applications, is application specific. Therefore, various objectives need to be compromised to achieve an optimal design. Various SRM design aspects have been studied and reported in the literature [1]–[4], emphasizing the parameters, the number of phases, the number of machine repetitions, and a few design ratios (the ratios between various internal dimensions of the machine). However, a comprehensive design methodology is nonexistent in the literature. The present research describes a design methodology and sets design guidelines to encompass the effects of machine geometry, configuration, and parameters on the overall performance of the SRM. Torque ripple, acoustic noise, and wide speed range are considered as critical issues during the design process. The existing body of SRM design knowledge has been utilized whenever possible. The proposed methodology has been applied to design a four-phase 8/6 (single repetition) 1.0-kW SRM. The overall performance of the designed SRM has been verified with static and dynamic system-level simulations. Finite-element analysis (FEA) has also been carried out to verify the static characteristic curves of flux linkage and torque.

## II. DESIGN METHODOLOGY

### A. Design Steps

Fig. 1 shows a comprehensive flowchart describing the design steps that constitute the SRM design methodology. These steps include the initial selection of the SRM configuration, parameters and design ratios, validation of an “output checklist,” verification of overall performance, and FEA. The design specifications may include all or some of  $T_{rated}$ ,  $\omega_b$ ,  $\omega_{mrate}$ ,  $P_{max}$ ,  $Pf$ ,  $V_{bus}$ ,  $I_{php}$ , torque ripple, acoustic noise, and ambient temperature, depending on the application. The selection of configuration encompasses selecting  $N_{ph}$ ,  $N_{rep}$ ,  $n_{ser}$ , and  $n_{par}$ . The parameters  $T_s$ ,  $T_{e\rho}$ ,  $J_{rms}$ , and  $B_{sat}$  are the major contributors to the envelope sizing and temperature rise of the machine. The design ratios are associated with specific requirements of mean torque, torque ripple, acoustic noise, and efficiency.

Although a complex SRM model [5] is used for static and dynamic performance analysis, simplified output equations are used for the initial sizing and “output checklist” to simplify the back and forth adjustment of the design variations. Configuration, parameters, and design ratios are changed within their own limits to obtain a satisfactory “output checklist” for the design. In the next step, static and dynamic performance of the candidate design are verified, ultimately using FEA, to check whether the critical machine parameters meet their requirements. The design steps are continued in an iterative manner until the design requirements are met. Most of the design iterations fall within the shaded blocks shown in Fig. 1, where the forward and backward design equations and guidelines are used.

Other inputs to the design process are the design ratios in the Nomenclature. These design ratios are ultimately determined by a combination of experience and more detailed analysis, some of which is given below and some of which is in the literature [5]–[12].

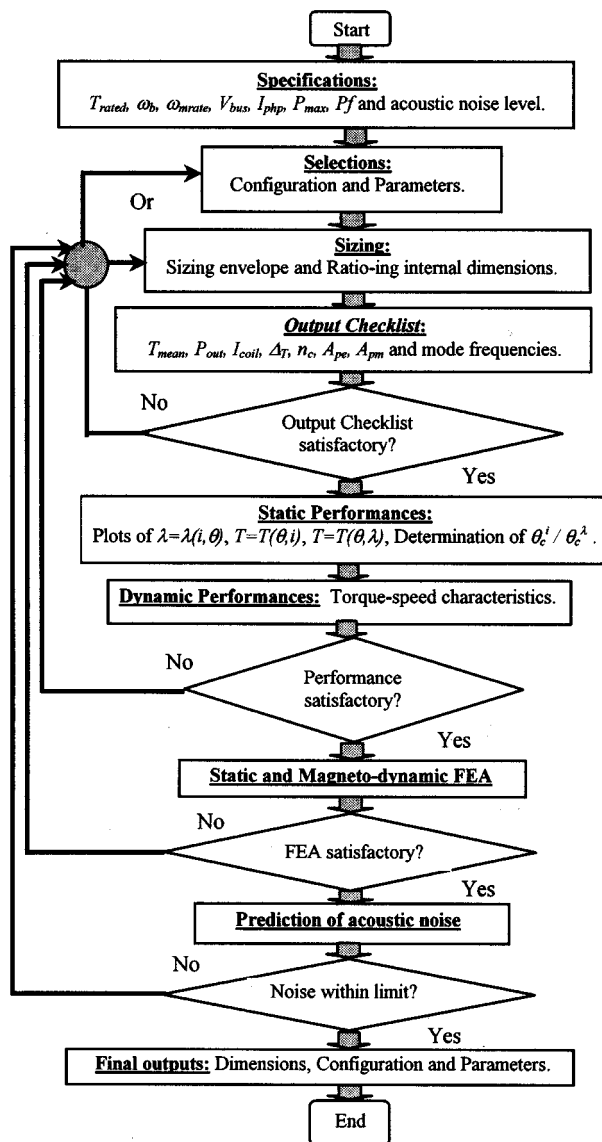


Fig. 1. Flowchart of the overall design methodology of SRM.

### B. Specifications for a Design Example

A four-phase 8/6 1.0-kW SRM is to be designed for an application requiring minimum acoustic noise, wide speed range operation, and minimum torque ripple. The specifications and torque–speed requirements for this design example are given in Table I. The general geometry of the machine is shown in Fig. 2. All of the plots given in the following sections are for this design example.

## III. SELECTION OF PARAMETERS AND CONFIGURATION

### A. Hoop Stress and Critical Speed

The concentrated stress due to a hole in a plane sheet is called the hoop stress. The rotor lamination ring can be treated as a plane sheet having a hole in the middle. The centrifugal forces at extremely high speed cause hoop stress in the rotor yoke. This is a tensile stress in a direction tangential to the circumference of the lamination ring to resist the strain causing the ring to lift off the shaft as shown in Fig. 3. During the design, it must be

TABLE I  
SPECIFICATIONS OF THE FOUR-PHASE 8/6 DESIGN EXAMPLE SRM

Speed (rad/sec and (rpm))	Torque (N.m)	Battery specifications	Dimensions & Temperature
0	3.0	Supply voltage	$L_{stk} = 0.0508$ m and $R_{out} = 0.0381$ m.
$\omega_b = 157.1$ (1500)	3.0	$V_{bus} = 42$ volts;	
261.8 (2500)	2.5	Battery current	All at ambient temperature $90^\circ$ C.
523.6 (5000)	1.5	$I_{php} \leq 40$ amps and	
$\omega_{mrate} = 837.76$ (8000)	0.5	Total external resistance = $0.1 \Omega$ .	
1570.8 (15000)	0.1		

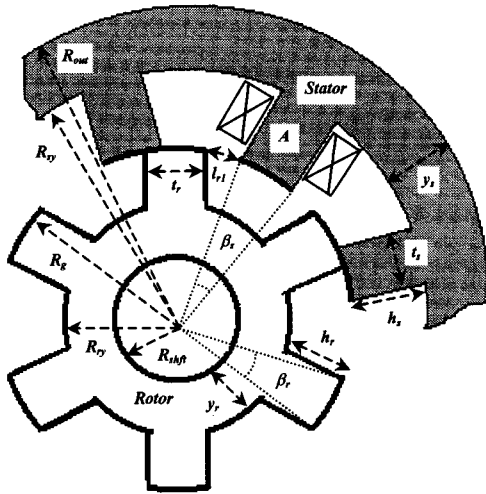


Fig. 2. A four-phase 8/6 SRM geometry where the rotor poles are in unaligned position with phase A.

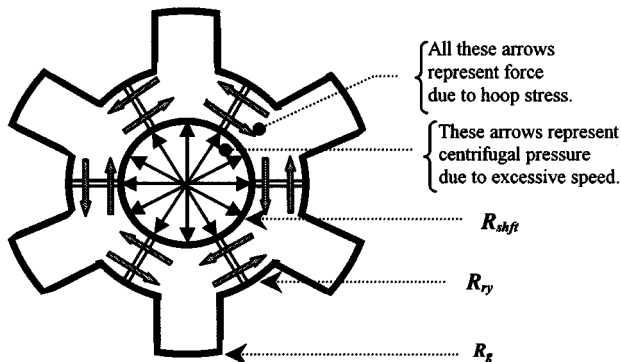


Fig. 3. Rotor lamination ring of a four-phase 8/6 SRM showing hoop stress.

insured that the hoop stress generated in the rotor lamination rings is sufficiently less than the rotor iron's yield strength. The rotor hoop stress  $T_s$  and maximum rated speed  $\omega_{mrate}$  of the machine determine the maximum allowed value of  $R_g$  as

$$R_g = \frac{1}{\omega_{mrate}} \sqrt{\frac{T_s}{K_h}}. \quad (1)$$

The hoop stress constant  $K_h$  is a function of the material density, its Poisson's ratio, and some design ratios [5].

The rotor's first critical speed  $n_c$  is higher with larger  $R_{shft}$  and with lower rotor weight and stack length, because they maximize the lateral stiffness of the rotor [6]. This speed is to be kept well above the maximum rated speed. Fig. 4 shows  $R_g$ ,  $L_{stk}$ , the product of  $R_g$  and  $L_{stk}$  (rotor envelop dimen-

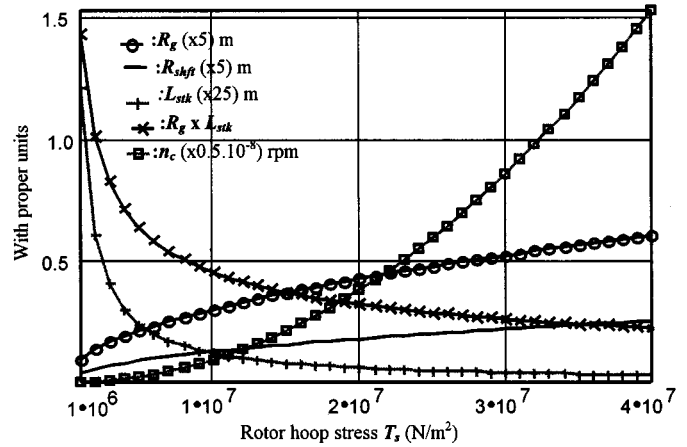


Fig. 4. Plots of  $R_g$ ,  $L_{stk}$ , product of  $R_g$  and  $L_{stk}$ ,  $R_{shft}$  and  $n_c$  with  $T_s$ .

sions),  $R_{shft}$ , and  $n_c$  as a function  $T_s$  when other parameters ( $\omega_b$ ,  $P_{max}$ ,  $T_{rated}$ ,  $A_{pm}$ ,  $A_{pe}$  and  $A_{wn}$ ) remain constant. From these plots, it is clear that envelope dimensions and volume of the machine decrease as  $T_s$  increases. It can be shown that the rotor power density increases as the square root of the rotor iron's yield strength [5]. Because the rotor envelope ( $R_g \times L_{stk}$ ) decreases more slowly as  $T_s$  is increased beyond a certain limit, it is possible to design a high-speed high-power machine by increasing  $L_{stk}$  and decreasing  $R_g$  for a reduced rotor hoop stress. The setting of the value of  $T_s$  depends on the design tradeoff between the rotor envelope dimensions and the first critical speed in order to optimize the operating speed and the power density.

Once all of the SRM dimensions have been chosen, the effective hoop stress  $T_{eh}$  can be recalculated in the "output checklist" as

$$T_{eh} = \frac{\rho_m \cdot \left[ 2(R_g - R_{ry}) \cdot R_g \sin\left(\frac{\beta_r}{2}\right) + \frac{\pi}{N_{rep}} (R_{ry}^2 - R_{shft}^2) \right]}{4 \cdot \cos\left(\frac{\pi}{2 \cdot (N_{ph} - 1)N_{rep}}\right) \cdot y_r \cdot (R_g + R_{shft})^{-1} \cdot \omega_{mrate}^{-2}}. \quad (2)$$

To ensure that the hoop stress of the designed machine is within limit, the value of  $T_{eh}$  from (2) should be less than or equal to the initial hoop stress  $T_s$  used in (1).

### B. Rotor Torque Density

An important parameter is the rotor torque density  $T_{ep}$  (torque per unit rotor volume), which depends strongly on the  $B_{sat}$  of the iron used. Because  $T_{ep}$  depends on the type of enclosure and the cooling process employed in a particular application, its value can vary from  $2.5 \text{ kN}\cdot\text{m}/\text{m}^3$  to  $250 \text{ kN}\cdot\text{m}/\text{m}^3$  [6]. Without any forced cooling, the value for  $T_{ep}$  is about  $100 \text{ kN}\cdot\text{m}/\text{m}^3$  for SRMs employing cobalt-iron ( $\text{CoFe}$ ) and  $52 \text{ kN}\cdot\text{m}/\text{m}^3$  for SRMs using silicon-steel ( $\text{SiFe}$ ). The rotor torque density varies inversely with the stack length as

$$L_{stk} = \frac{P_{max}}{\pi \cdot \omega_b \cdot R_g^2 \cdot T_{ep}}. \quad (3)$$

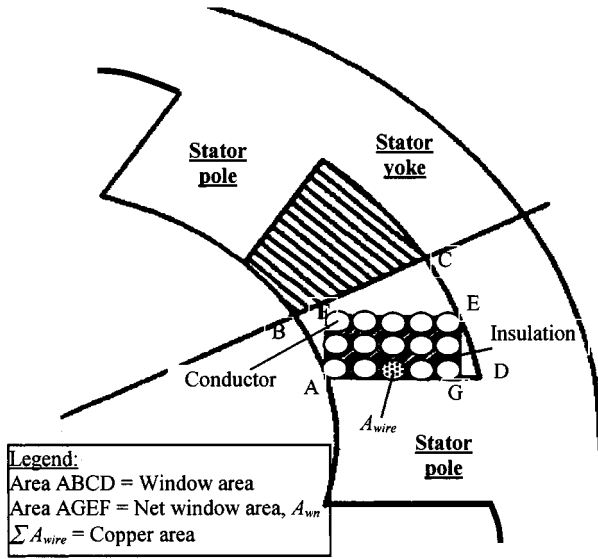


Fig. 5. Partial cross-sectional view of two adjacent stator poles showing the slot fill for phase windings.

In the design iterations of Fig. 1, the value of the mean rotor torque density  $T_{\text{density}}$  recalculated by (4) should be greater than or equal to  $T_{e\rho}$

$$T_{\text{density}} = K_t \cdot \frac{J_{\text{rms}} \cdot A_{\text{wn}} \cdot B_{\text{sat}} \cdot S_{t f} \cdot n_{\text{ser}} \cdot n_{\text{par}}}{R_g} \quad (4)$$

$K_t$  is a constant containing the ratios between the peak and rms values of phase current and torque, given by

$$k_t = \frac{\text{Peak phase current/Rms phase current}}{\pi \cdot \text{Peak phase torque/Average total torque}}$$

The peak to rms ratios are determined from detailed simulations of the proposed design. The constant  $k_t$  does not have a large variation and is typically equal to about 0.59.

### C. RMS Current Density in the Coils

The rms current density  $J_{\text{rms}}$  is an important design parameter that affects a number of design outputs. The area products  $A_{\text{pm}}$  and  $A_{\text{pe}}$ , representing mechanical and electrical loading of the machine, obey the design constraint  $A_{\text{pm}} \geq A_{\text{pe}}$  [5]. The expressions for these area products are

$$A_{\text{pe}} = \frac{8 \cdot \pi \cdot \alpha_{\text{rms}} \cdot P_{\text{max}}}{S_{t f} \cdot (1 - \alpha_u) \cdot P f \cdot \eta \cdot B_{\text{sat}} \cdot J_{\text{rms}} \cdot K \cdot \omega_b} \quad (5)$$

$$A_{\text{pm}} = 4 \pi \cdot R_g \cdot L_{\text{stk}} \cdot N_{\text{rep}} \cdot N_{\text{ph}} \cdot A_{\text{wn}} \quad (6)$$

Fig. 5 defines the net window area  $A_{\text{wn}}$  and the window factor  $K$ , which is the ratio between the window area and the copper area. The stacking factor  $S_{t f}$  is the ratio between actual stator length and the effective length of the iron. The actual length of the stator is greater than the length of the iron due to space between the stacked laminations and the insulation (often oxide) on the laminations.

The efficiency  $\eta$  used in (5) to calculate  $A_{\text{pe}}$  accounts for the iron loss of the machine. If the iron loss is high, the efficiency becomes low and, consequently,  $A_{\text{pe}}$  increases. This eventually will require a higher machine geometry to fulfill the

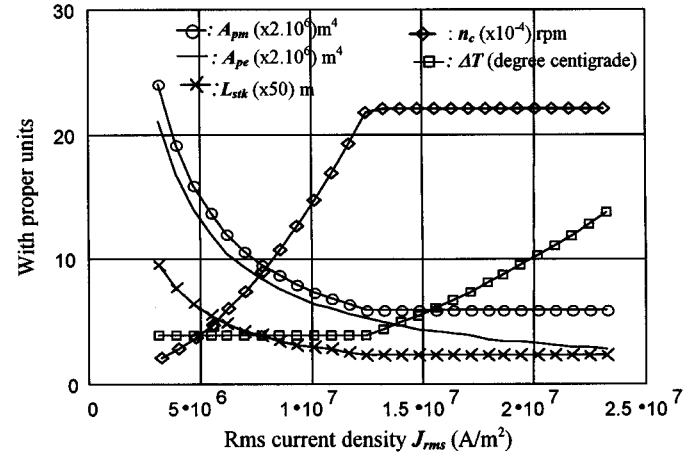


Fig. 6. Plots of  $A_{\text{pe}}$ ,  $A_{\text{pm}}$ ,  $L_{\text{stk}}$ , and  $\Delta T$  with rms current density  $J_{\text{rms}}$ .

design constraint of  $A_{\text{pm}} \geq A_{\text{pe}}$ . Fig. 6 shows the effect of  $J_{\text{rms}}$  on  $L_{\text{stk}}$ ,  $A_{\text{pm}}$ ,  $A_{\text{pe}}$ ,  $\Delta T$ , and  $n_c$ . In this figure, the parameters  $T_s$ ,  $T_{e\rho}$  and the ratio of iron to copper area in the windows are held constant. The value of  $L_{\text{stk}}$  must satisfy both the back electromotive force (BEMF) and maximum torque density requirements of the machine. The value for  $A_{\text{pm}}$  is always higher than  $A_{\text{pe}}$ , for any value of  $L_{\text{stk}}$  [5]. However,  $A_{\text{pm}}$  starts diverging from  $A_{\text{pe}}$  when  $L_{\text{stk}}$  attains its minimum value and the critical speed  $n_c$  reaches its maximum. For an optimal utilization of mechanical and electrical loading of the geometry,  $J_{\text{rms}}$  is chosen to be somewhere near this divergence subject to the constraint that  $J_{\text{rms}}$  is not so large as to cause excessive copper losses. The values for optimal  $A_{\text{pe}}$  and  $A_{\text{pm}}$  are determined by trial and error to get the required machine performance. Fig. 6 also shows the dependence of the allowable maximum rms current density on  $\Delta T$ , which is a function of thermal conductivity of the coil sides, shape of the conductor, and cooling process of the wires [6]. If the rise of  $\Delta T$  above the ambient is higher than the insulation limit, the loading inequality  $A_{\text{pm}} \geq A_{\text{pe}}$  is maintained by increasing the SRM's envelop dimensions by decreasing  $J_{\text{rms}}$  in the coils.

The other parameters associated with  $J_{\text{rms}}$  are the mean torque density  $T_{\text{density}}$ , the number of turns  $N_p$ , and the peak phase current  $I_{\text{plp}}$ . The variations of these parameters with  $J_{\text{rms}}$  are shown in Fig. 7, where the window areas of coil sides have been kept fixed. As expected,  $T_{\text{density}}$  increases with an increase in  $J_{\text{rms}}$ . Although  $I_{\text{plp}}$  remains the same,  $N_p$  must increase to maintain higher ampere-turns (magnetomotive force) with the increase of  $J_{\text{rms}}$ . With a lower  $J_{\text{rms}}$ ,  $T_{\text{density}}$  as well as mean rotor torque are maintained by increasing the envelope dimensions without violating the loading inequality constraint.

### D. Number of Phases and Poles per Phase

The selection of  $N_{\text{ph}}$  and  $N_{\text{rep}}$  depends primarily on the application. The multiplicity of the basic SRM configuration  $N_{\text{rep}}$  is equal to the number of pole pairs per phase.  $N_{\text{ph}}$  and  $N_{\text{rep}}$  together set the numbers of stator and rotor poles. Having a higher number of rotor poles gives a smaller stroke angle and leads to lower torque ripple at the cost of a decreased saliency ratio (ratio

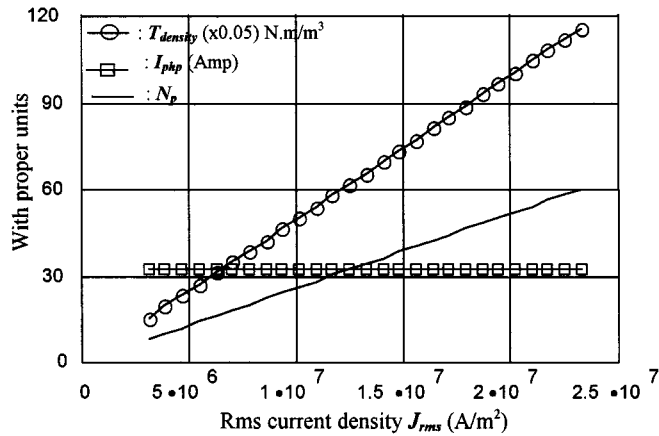


Fig. 7. Plots of  $T_{density}$ ,  $I_{php}$  and  $N_p$  with rms current density  $J_{rms}$ .

between the maximum and minimum inductance levels). The decrease in saliency ratio will increase the controller volt-amperes and decrease the torque output. The number of strokes per revolution can be increased with a higher number of phases to alleviate the problem of torque dips with a smaller penalty in the saliency ratio [6]. Because of the lower torque dips, the average torque of the machine will increase. It would appear that doubling  $N_{rep}$  (while other parameters are held constant with all the coils of each phase connected in series) doubles the angular rate of change of phase flux, resulting in doubled torque at a given speed and current since the maximum and minimum inductances are unchanged. However, to a first order the total torque does not depend on the number of repetitions because  $N_p$  will have to be halved to keep the BEMF the same when  $N_{rep}$  is doubled. Further consideration of the rate of change of flux linkage, available coil area, saliency ratio, split ratio ( $\alpha_{RgRout}$ ), variation in the magnetic circuit reluctance, saturation behavior, and the iron loss due to the increase of the repetition modifies this simplistic conclusion. Doubling the number of repetitions will not contribute to the mean torque produced unless the pole width is made more than 50% of that for a single repetition machine. To maximize the co-energy both under high- and low-current conditions, the optimized stator pole width for a two-repetition machine is found to be approximately 70% of that of the single-repetition one [3]. This gives about 40% more thermally limited torque and horizontal force for the same copper loss and active volume. The above dependency of mean torque and torque dip with  $N_{ph}$  and  $N_{rep}$  can be explained with  $K_L$ , which is the ratio of inductance overlap of two adjacent phases to the angle over which the inductance is changing [1]. Equation (7) and Fig. 8 together define  $K_L$

$$K_L = 1 - \frac{\text{Stroke Angle}}{\min(\beta_s, \beta_r)}$$

where

$$\text{Stroke Angle} = \frac{\pi}{N_{ph} \cdot N_{rep} \cdot (N_{ph} - 1)}. \quad (7)$$

The rotor pole arc  $\beta_r$  can be made slightly larger than the stator pole arc  $\beta_s$  to accommodate some of the fringing stator fluxes. However, making either the rotor or stator poles wider

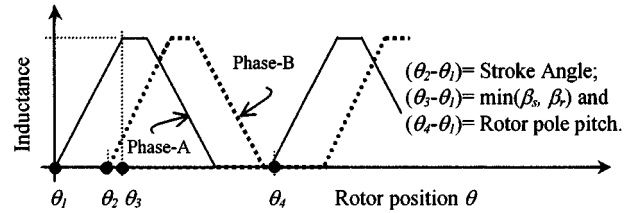


Fig. 8. Variation of phase inductance with rotor position.

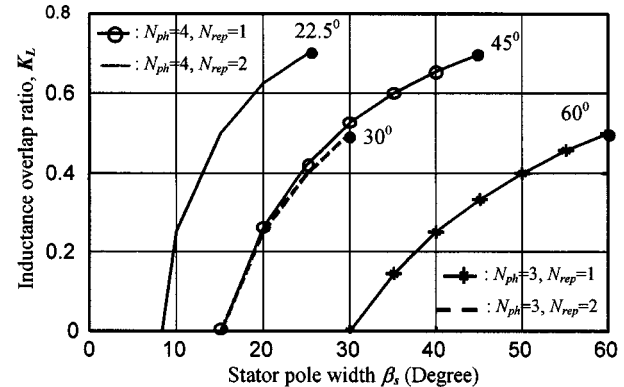


Fig. 9. Inductance overlap ratio versus stator pole width.

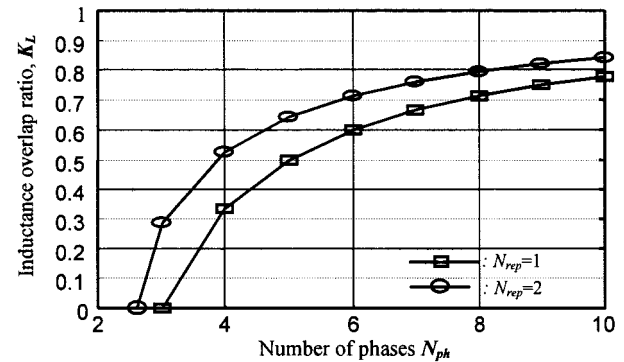


Fig. 10. Inductance overlap ratio versus number of phases.

than the other takes torque away from the aligned position. When one pole is wider than the other, there is a region around the aligned position where the stored energy (and inductance) does not change and, thus, the torque is zero. Thus, the pole arcs should be wide enough to insure that there is torque production for all rotor positions. Figs. 9 and 10 plot  $K_L$  versus  $\beta_s$  (assuming  $\beta_s \leq \beta_r$ ) and  $K_L$  versus  $N_{ph}$  (assuming  $\beta_s$  for  $N_{rep} = 2$  to be 70% of that of  $\beta_s$  for  $N_{rep} = 1$ ). The higher the  $K_L$ , the lower will be the torque dip and the higher will be the mean torque as well. Fig. 9 shows that the minimum stator pole widths to start the machine (for  $K_L \geq 0$ ) are approximately  $10^\circ$  and  $30^\circ$  for a “ $N_{ph} = 4, N_{rep} = 2$ ” and a “ $N_{ph} = 3, N_{rep} = 1$ ” machine, respectively. Therefore, it can be concluded that higher values of  $K_L$  are achievable at relatively low values of  $\beta_s$  for a machine with more phases and/or repetitions. These same machines have better starting capabilities. Figs. 9 and 10 also show that the improvement upon the problem of torque dip is noticeable in the lower range of  $\beta_s$  and lower  $N_{ph}$ . Thus, the number of phases or the

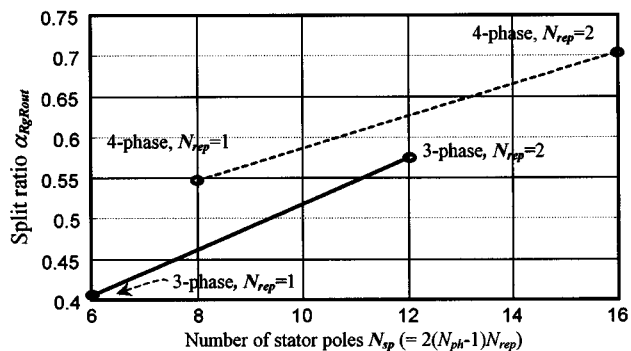


Fig. 11. Variation of split ratio with the change of  $N_{ph}$  and  $N_{rep}$ .

stator pole width should not be increased too much, since an increased number of phases increases the converter cost and an increased stator pole width decreases  $A_{wn}$ .

Another perspective on selecting  $N_{ph}$  and  $N_{rep}$  is that the increase in  $N_{ph}$  and  $N_{rep}$  requires a smaller pole width and, thus, a smaller stator back-iron width. Consequently,  $\alpha_{RgRout}$  can be increased as shown in Fig. 11 while keeping the other design ratios and envelope dimensions constant. A higher  $\alpha_{RgRout}$  provides space for a larger rotor diameter within the given frame size, which, in turn, produces more torque.

The higher the  $N_{rep}$  and  $N_{ph}$  of the machine, the less time there is available for the rise and fall of the phase currents. It is true that if  $N_{ph}$  or  $N_{rep}$  is doubled, the time for the phase current to rise is halved; fortunately, the current to be reached is also halved. Thus, unless the inductance has increased, there will be no change in the total power due to all of the phases. It turns out that the BEMF does not depend on the pole width so that the total number of turns is the same ( $N_p$  decreases if pole windings are in series), even if  $N_{ph}$  is doubled. The unaligned inductance stays the same then because the path length halves, but so does the area of the coil sides. The two effects cancel leaving the unaligned inductance the same since the number of turns are the same. Thus, there is no more difficulty reaching the required (1/2) phase current with twice as many phases or repetitions if and only if the pole width with twice as many stator poles is halved. If the pole width has not been halved for the two-repetition machine, then it is harder to get the current into the machine and lower performance results.

Machines with  $N_{rep} = 2$  have shorter flux paths, which reduce the core losses and the absorption of magnetomotive force (MMF) in the stator yoke. Shorter paths are achieved by means of the winding configuration and the lamination geometry. A higher inductance ratio in machines with higher phase numbers is also advantageous for sensorless operation.

#### IV. SIZING OF ENVELOPE AND INTERNAL DIMENSIONS

##### A. Envelope Sizing

The selection of envelope dimensions ( $R_g$  and  $L_{stk}$ ) is governed by (1)–(6) in Section III. All other stator and rotor dimensions are determined as functions of  $R_g$  and the following four basic design ratios.

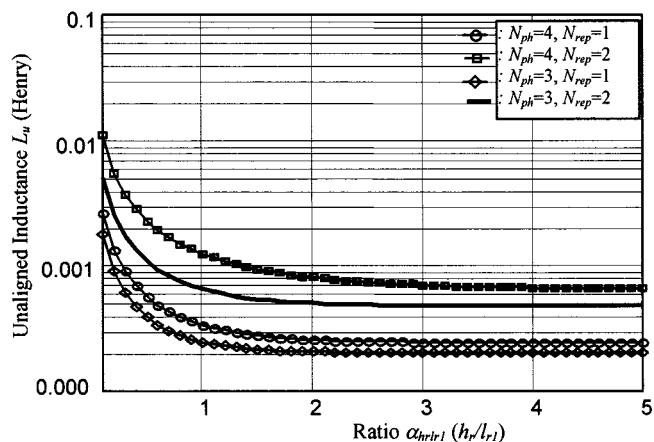


Fig. 12. Unaligned inductance versus the ratio  $\alpha_{hrtr1}$ .

TABLE II  
LIST OF FOUR BASIC DESIGN RATIOS FOR THREE- AND FOUR-PHASE SRMs

Design Ratios	$N_{ph}=4, N_{rep}=1$	$N_{ph}=4, N_{rep}=2$	$N_{ph}=3, N_{rep}=1$	$N_{ph}=3, N_{rep}=2$
$\alpha_{hrtr1}$	2.0~2.5	2.5~3.0	1.5~2.0	2.0~2.5
$\alpha_{hrhs}$	0.65~0.7	0.65~0.7	0.65~0.7	0.65~0.7
$\alpha_n$	0.5~0.67	0.5~0.67	0.5~0.67	0.5~0.67
$\alpha_{yrys}$	0.75~0.85	0.75~0.85	0.75~0.85	0.75~0.85

##### B. Design Ratios

The design ratios are defined to be the ratios between various internal dimensions of the machine. The objective functions of saliency ratio, unaligned inductance, torque, torque ripple, torque per ohmic loss, efficiency, acoustic noise, and torque–speed range [4]–[7] are taken into consideration in choosing the design ratios. A good understanding of the MMF profiles inside the machine geometry helps significantly in choosing the appropriate design ratios. The four basic design ratios are  $\alpha_{hrtr1}$ ,  $\alpha_{hrhs}$ ,  $\alpha_n$  and  $\alpha_{yrys}$  as defined in the Nomenclature.

The ratio  $\alpha_{hrtr1}$  is chosen based on the desired saliency ratio and unaligned inductance  $L_u$ . Fig. 12 shows  $L_u$  versus  $\alpha_{hrtr1}$  for 3 and 4-phase SRM's. The higher the ratio  $\alpha_{hrtr1}$ , the lower is the  $L_u$ , which gives a higher saliency ratio. This ratio need not be greater than a certain maximum, since  $L_u$  levels off after that maximum. The objectives for selecting optimal values for  $\alpha_n$ ,  $\alpha_{hrhs}$  and  $\alpha_{yrys}$  are to maximize the stator mode frequencies, to minimize the envelope dimension (product of  $R_{out}$  and  $L_{stk}$ ), to contain  $\alpha_{RgRout}$ ,  $\alpha_{tsP}$  and  $\alpha_{hrtr1}$  within limits and to insure a critical speed  $n_c$  well above the rated speed  $\omega_{mrated}$  [7]. Table II gives a list of these design ratios for three- and four-phase SRMs.

##### C. Sizing $h_r$ and $h_s$

The values of  $h_r$  and  $h_s$  are calculated using the following equation after choosing  $\alpha_{hrtr1}$  and  $\alpha_{hrhs}$  from Table II:

$$h_s = 2.R_g \cdot \frac{\alpha_{hrtr1}}{\alpha_{hrhs}} \cdot \sin \left( \frac{\pi}{4(N_{ph} - 1)N_{rep}} - \frac{\beta_r}{2} \right). \quad (8)$$

#### D. Sizing $R_{\text{out}}$

Outer radius  $R_{\text{out}}$  is calculated from  $R_g$  and  $h_s$  as

$$R_{\text{out}} = h_s + (R_g + g) \cdot \left( 1 + 2 \cdot \alpha_n \cdot \sin \left( \frac{\beta_s}{2} \right) \right). \quad (9)$$

It is important to note here that, if a maximum  $R_{\text{out}}$  is given as a design constraint, the ratio  $\alpha_{RgR_{\text{out}}}$  should be as high as possible to provide larger rotor diameter within the given frame to produce more torque. However, this will shrink the available window area for the windings, thus reducing the available MMF of the machine.

#### E. Sizing $t_r$ and $t_s$

Dimensions  $t_r$  and  $t_s$  are the lateral length of the rotor and stator pole arcs, respectively. The rotor pole arc  $\beta_r$  should be slightly larger than the stator pole arc  $\beta_s$  to ensure the production of torque at all rotor positions without sacrificing the aligned inductance. This is achieved by allowing  $t_r$  to exceed  $t_s$  by a length between  $g$  and  $2g$  [6].

#### F. Sizing $y_s$ and $y_r$

In an SRM with a two-pole flux pattern, the main flux divides into two equal parts when it leaves the poles and enters the yoke. Again, the sections of the yoke are shared between different overlapping phases, especially during high-speed operation. Hence,  $y_r$  and  $y_s$  should be more than 50% of  $t_r$  and  $t_s$ , respectively, to prevent back-iron saturation. The ratio of  $y_s$  over  $t_s$  ( $\alpha_n$ ) is also important from an acoustic noise point of view. The other consideration for choosing  $y_s$  is the machine weight limit. Dimension  $y_s$  is calculated as

$$y_s = R_{\text{out}} - \left[ R_g + g + 2 \cdot R_g \cdot \frac{\alpha_{hr} t_r 1}{\alpha_{hr} h_s} \cdot \frac{\sin \left( \frac{\pi}{4(N_{\text{ph}} - 1)N_{\text{rep}}} - \frac{\beta_r}{2} \right)}{\cos \left( \frac{\beta_s}{2} \right)} \right]. \quad (10)$$

### V. CRITICAL ISSUES

#### A. Acoustic Noise [7]

It is desirable to minimize the acoustic noise in SRM drives for many applications to avoid environmental noise pollution. The design objectives for noise reduction are to make the natural mode frequencies of the stator geometry as high as possible and to decrease the harmonic components of the magnetic radial force acting on the stator. A good design can be achieved if the dominant mode frequencies can be made higher than the audible range frequencies. The design ratios  $\alpha_n$  and  $\alpha_{hr} h_s$ , in Table II, need to be increased to have a thicker stator yoke to increase the mode frequencies. A thicker yoke reduces the SRM's power density, resulting in a lower utilization of the iron. Furthermore, if  $R_{\text{out}}$  is constrained, the net available window area

TABLE III  
LIST OF  $\alpha_n$  AND  $\alpha_{hr} h_s$  FOR LOW ACOUSTIC NOISE SRM

Design Ratios	$N_{\text{ph}}=4, N_{\text{rep}}=1$	$N_{\text{ph}}=4, N_{\text{rep}}=2$	$N_{\text{ph}}=3, N_{\text{rep}}=1$	$N_{\text{ph}}=3, N_{\text{rep}}=2$
$\alpha_n$	1.0~1.3	1.0~2.0	1.0~1.2	1.0~1.5
$\alpha_{hr} h_s$	1.0~1.5	0.7~1.2	1.0~1.5	0.7~1.2

TABLE IV  
DESIGN RATIOS  $\alpha_{tsP}$  AND  $\alpha_{RgR_{\text{out}}}$ , TORQUE AS AN OBJECTIVE FUNCTION

Objective function	$\alpha_{tsP}$	$\alpha_{RgR_{\text{out}}}$
<b>Torque</b>	0.27~0.47	0.57~0.63

for the windings decreases and the thermal diffusion distance for heat transfer from the windings to the outside frame increases. Therefore, the design choice for a low-noise machine requires a tradeoff among the desired quantities. The ranges for  $\alpha_n$  and  $\alpha_{hr} h_s$ , optimized for low acoustic noise design, are given in Table III.

The choice of the stator pole shape can play a small but effective role in reducing acoustic noise. A method known to be effective is to taper the stator pole from the top towards the air gap and to round the bottom of the slot. Analysis also shows that the greater the number of phases or repetitions, the lower the acoustic noise power level.

#### B. Torque Ripple

The application of an SRM in a servo system requires reduced torque ripples. One method of reducing torque ripple is to increase  $N_{\text{ph}}$  and  $N_{\text{rep}}$ , as explained in Section III-D. Increasing  $\beta_r$  and  $\beta_s$  also reduces torque deadband between adjacent phases and, thus, reduces torque ripples, assuming a suitable control design is used [2]. The dynamic torque ripples of the machine are reflected in the amount of torque dip found in the static plots of torque versus rotor position, as shown in Fig. 15. Reducing the torque dip may result in a lower mean torque, and a compromise solution must be sought. The ratios  $\alpha_{tsP}$  and  $\alpha_{RgR_{\text{out}}}$  should lie within the range given in Table IV [4] in order to maximize the mean torque. In the design iterations of Fig. 1, these two ratios are checked in the "output checklist" evaluation.

#### C. Wide Speed Range [8]

A wide speed range with adequate torque capability at higher speeds is desired in electric-propulsion-type applications. An SRM can satisfy this requirement only with efficient torque sharing between phases, which can be determined by two critical rotor positions. The critical rotor positions  $\theta_c^i$  and  $\theta_c^\lambda$  represent the two crossover angles where two neighboring phases produce equal values of torque at equal values of current and flux linkage, respectively. The design strategy is to extend these rotor positions, enabling the controller with dwelling flexibility. The SRM drive thus obtained will have wide speed range, lowered torque ripple, and extended constant-power range of operation. The higher the ratios  $\alpha_{y_r y_s}$  and  $\alpha_{hr} h_s$ , the wider is the separation between  $\theta_c^i$  and  $\theta_c^\lambda$ . The same objective can also be achieved by making the air-gap length as small as manufac-

TABLE V  
DIMENSIONS, CONFIGURATION, PARAMETERS, AND SOME OUTPUTS  
OF THE DESIGNED FOUR-PHASE 8/6 SRM

Dimensions (m)	Configuration/Parameters	Outputs
$L_{stk}=0.0508$ ; $R_{shaft}=0.009$ ; $R_{ry}=0.013$ ; $R_g=0.0203$ ; $R_{sy}=0.0305$ ; $R_{out}=0.038$ and $g=0.000254$ $\beta_s=22.5^\circ$ ; $\beta_r=22.8^\circ$	$n_{par}=1$ ; $n_{ser}=2$ ; $N_p=35$ $B_{sat}=1.8$ Tesla (M19 SiF <sub>e</sub> ) $A_{wire}=7.1 \cdot 10^{-7}$ m <sup>2</sup> $W_r=1.135$ kg $\theta_c^\lambda = -13^\circ$ ; $\theta_c^i = -9^\circ$	$\Delta T=6.5^\circ$ C; $R_{ph}=0.3$ $\Omega$ ; $n_c=2 \cdot 10^5$ rpm; $f_{m(-0)} = 14200$ Hz $f_{m(-2)} = 1600$ Hz $f_{m(-4)} = 7961$ Hz Mean Noise=36.75 dB

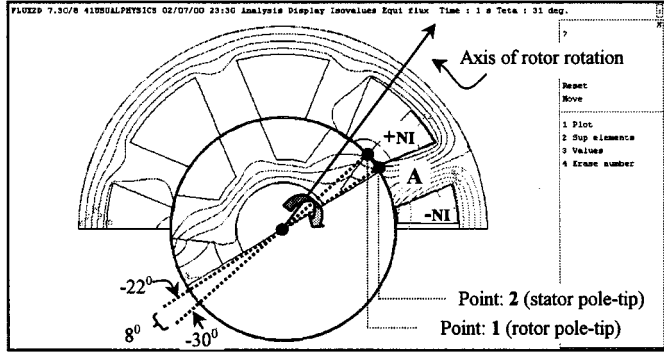


Fig. 13. Geometry of the designed four-phase 8/6 SRM.

turally possible. This criterion matches with the criteria to maximize the SRM's power density and torque density. Changing  $\beta_s$ ,  $B_{sat}$  and  $N_p$  have conflicting effects on  $\theta_c^i$  and  $\theta_c^\lambda$ , which suggests that these parameters should be selected together in order to optimize the overall performance of the machine.

### VI. FINAL OUTPUTS OF THE DESIGN EXAMPLE

The final results for the design example are summarized in Table V. This is a four-phase 8/6 (single repetition) 1-kW SRM as shown in Fig. 13.

#### A. Static Performances

Using the dimensions, configuration, parameters, and outputs from Table V, the static plots of flux linkage ( $\lambda$ ) and torque ( $T$ ) are checked to insure that they are satisfactory. Fig. 14 shows the  $\lambda$ - $i$ - $\theta$  characteristic curves for the designed SRM calculated both by the analytical and the FEA methods. The maximum flux linkage at the pole-aligned position reaches a value of about 0.05 W at the maximum  $I_{php}$  of 40 A. This gives a saturation flux density of 1.89 T, whereas the  $B_{sat}$  for the M19 SiF<sub>e</sub> used for this SRM is 1.8 T. Therefore, the machine geometry is utilizing the core material maximally up to its saturation level.

Fig. 15 shows the  $T$ - $i$ - $\theta$  characteristic curves for the same SRM. Both the analytical and the FEA methods are employed to generate torque profiles for two adjacent phases. The maximum torque is about 3 N·m, which is sufficient to meet the torque requirement of the application.

The torque profiles of Fig. 15 show similar characteristics for the analytical and the FEA methods. The only significant difference in the characteristics is around the rotor position of  $-22^\circ$  where rotor pole tip (point: 1) reaches stator pole tip (point: 2) as shown in Fig. 13. At this position, the effective air gap steps

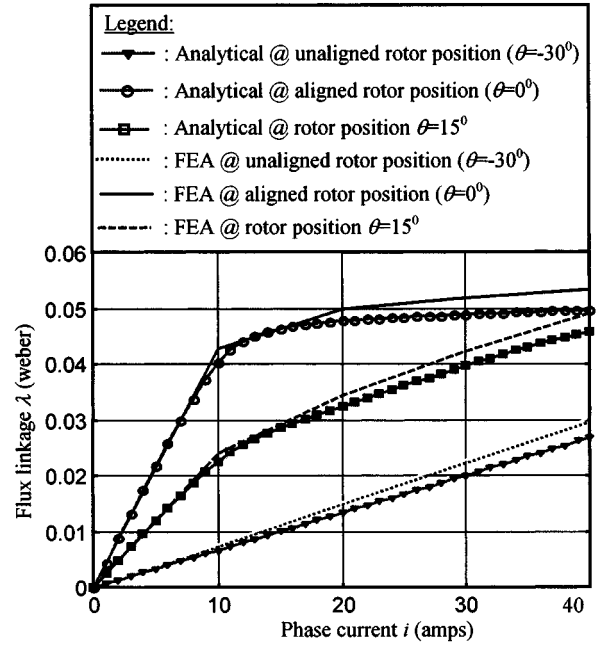


Fig. 14. Plots of flux linkage versus current for various rotor positions.

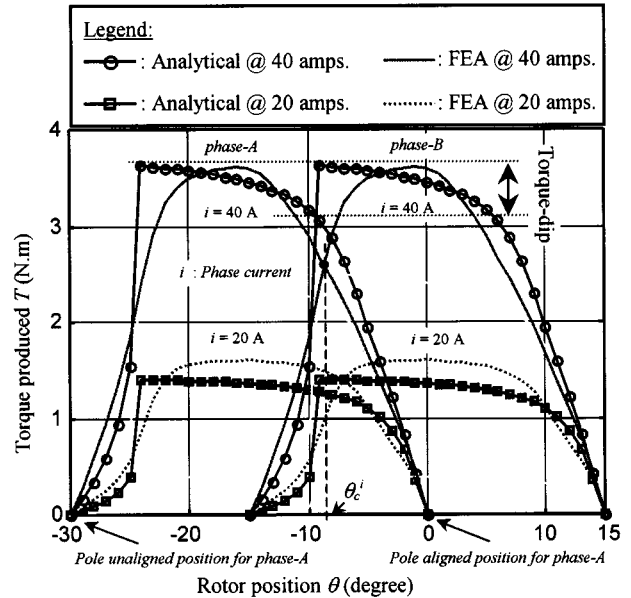


Fig. 15. Plots of torque versus rotor position for various current levels.

down from a higher value to  $g$ , which causes a discontinuity for the analytical calculation of static torque.

#### B. Dynamic Performance

System-level dynamic simulation to investigate the torque-speed characteristics of the designed SRM is done using the control algorithm described in [8]. The optimal phase commutation angle is varied from  $\theta_c^i$  and  $\theta_c^\lambda$  as a function of operating speed, torque, and phase current to maximize the torque-speed envelope. The computed torque-speed characteristic for the design example is shown in Fig. 16.

Comparing the torque-speed requirement in Table I and the computed characteristics in Fig. 16, it can be claimed that the

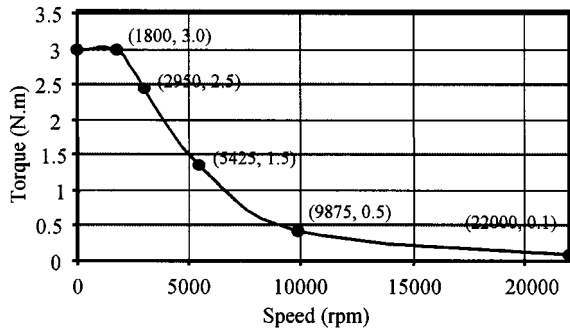


Fig. 16. Torque-speed characteristics of the designed SRM.

designed SRM is capable of handling the torque-speed requirements of the application.

## VII. CONCLUSIONS

A comprehensive design methodology is presented in this paper. The use of the methodology is illustrated with a design example. The critical issues of torque ripple, acoustic noise, and wide speed range are addressed in the design process. The results establish design guidelines that encompass the effects of machine geometry, material properties, configuration, design parameters, and design ratios on the overall performance of the SRM.

Design examples with particular attributes, such as low acoustic noise design or wide speed range SRM design, are discussed in separate papers to provide sufficient details [7], [8]. Experimental verification of the design equations and methodology presented here appear in related publications [9], [10]. The calculations of various losses during design will be presented in a future paper.

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